Computational Methods PhD Qualifying Exam

Fall, 2018

Mechanical Engineering, University of Arkansas

PROBLEM 1:

A. For the 2x2 matrix given below, find the eigen values and eigen vectors of the matrix.

$$\begin{bmatrix} -1 & 2 \\ 2 & 2 \end{bmatrix}$$

B. Diagonalize the 2x2 matrix in part (a) using the eigen values and eigen vectors.

C. Compute the determinant of the matrix below:

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

Note: Show all steps to reach the final answer in parts (a) (b) and (c).

PROBLEM 2:

Evaluate the following integral:

$$\int_0^3 (1 - e^{-x})dx$$

A. Analytically;

B. Single application of trapezoidal rule;

C. Single application of Simpson's 1/3 rule

D. Multiple application of trapezoidal rule with n = 2;

E. Compare the results from (a) to (d) and discuss the causes of the differences.

PROBLEM 3:

Consider the ODE below:

$$\frac{d^2y}{dx^2} + x^2\left(\frac{dy}{dx}\right)^2 + xy = 0$$

$$y(0) = 2,\ y(5) = 1$$

A. Write a set of first order ODEs and initial conditions if you were using **Euler Shooting method**.

B. Write the Excel VB functions one would use to solve the ODE using Euler Shooting method.

C. Write what you would enter in each shaded Excel cell below if you're using Euler Shooting Method with **h = 0.2** by referencing the VBA functions where necessary.

C3 = _____ D3 = _____ E3 = _____

F3 = _____ G3 = _____

H3 = _____ C4 = _____ D4 = _____

	A	B	C	D	E	F	G	H
1	**h =**	**0.2**						
2	i	x	y1	y2	k_{11}	k_{12}	$y_{1,i+1}$	$y_{2,i+1}$
3	1	0.0						
4	2	0.2						

PROBLEM 4:

A race car driver accelerates from a stop, and her position was recorded at different moments in time as given below.

time (s)	0.0	2.0	4.0	6.0	8.0
position (m)	0.00	10.43	34.81	85.33	140.79

A. With the given data, determine the best estimate of her **velocity** at t = 6 seconds. Use **two decimal places**.

B. With the given data, determine the best estimate of her **acceleration** at t = 0 seconds. Use **two decimal places**.

C. If her position actually matches the function $\mathbf{p(t) = 2.08t^2 + 1.22t - 0.42}$, determine the **true error** for your answer in part (b) [use **2 decimal places**].

PROBLEM 5:

An 80 kg sky-diver jumping out of an airplane on Earth, in free-fall, is modelled as a 1-D problem. Assume the downward force due to gravity is constant (with acceleration of 9.81 m/s^2) and that the velocity is zero at time equals zero. For the two following assumptions (A. and B. below), use a numerical method of your choice (state the name of method used) to calculate the downward velocity numerically for a free-fall time of 5 seconds at 1-second increments.

A. The upward force due to drag is proportional to velocity with a proportionality constant of 12 kg/s. To check your solution, compare your answer with the analytical solution at the end of the last iteration.

B. You've learned that the drag can also be modelled as proportional to the coefficient of drag times the dynamic pressure times the cross-sectional area (i.e. $D = C_D \times \left(\frac{1}{2}\rho V^2\right) \times S$). Assume $C_D = 1.2$, $\rho = 1.223$ kg/m^3, V = velocity, and S = 0.1 m^2.